Dedicated to Professor Bernhard Wunderlich on the occasion of his 65th birthday

# ON THE CORRELATION OF THE GLASS TRANSITION TEMPERATURE OF A COPOLYMER WITH ITS COMONOMER SEQUENCES

### H. Suzuki, N. Kimura and Y. Nishio

Department of Bioengineering, Nagaoka University of Technology, Nagaoka, Niigata-Ken 940-21, Japan

## Abstract

The glass transition temperature of a copolymer depends not only on chemical composition but also on its comonomer sequences. This experimental fact is explained by Barton's and Johnston's equations. Their equations, though complicated, become simple, if a suitable parameter is used to describe the comonomer sequences. It is shown that with these new expressions, their equations can be used to understand glass transition temperatures of two additional types of copolymers, compatible multiblock copolymers and homopolymers with various tacticities treated as steric copolymers.

Keywords: comonomer sequence, copolymer, glass transition temperature, homopolymer, tacticity

## Introduction

An ultimate objective for polymer scientists is to understand the structureproperty relationships of polymers. Here, we will consider a possible correlation of the glass transition temperature  $(T_g)$  of a copolymer with its chain structure. This means the sequences of two monomers, A and B, in a given copolymer.

There have been frequent attempts to elucidate the variation in  $T_g$  of a copolymer as a function of composition, as is shown in Fig. 1. This figure shows that the  $T_g$  of an alternating copolymer does not lie on a line expected for the corresponding statistical copolymers [1]. In other words, there exists a definite difference in  $T_g$  between two copolymers with the same composition but different comonomer sequences. This fact reveals a defect in linear equations such as

$$T_{\rm g} = m_{\rm A} T_{\rm gA} + m_{\rm B} T_{\rm gB} \tag{1a}$$

$$1/T_{g} = W_{A}/T_{gA} + W_{B}/T_{gB}$$
(1b)

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Fig. 1 Plot of  $T_g$  vs.  $m_{MMA}$  for styrene-methyl methacrylate (S/MMA) statistical (o) and alternating (•) copolymers and the two corresponding homopolymers ( $\Box$ ) [1]

Here,  $T_g$  and  $T_{gx}$  are the glass transition temperatures, respectively, of a given copolymer specified by mole fraction (*m*) or weight fraction (*W*) and of a homopolymer of monomer X (= A or B). One can see the necessity of introducing the second variable which describes the comonomer sequences.

More than twenty years ago, nonlinear equations addressing this problem were proposed by Barton [2] and Johnston [3]. Unfortunately, their equations were too complicated for practical use and have been applied only to understand the variation in  $T_g$  of statistical copolymers or to estimate the  $T_g$  of the alternating copolymer in a given system. However, if a suitable parameter is used to describe the comonomer sequences, their equations become quite simple. Such expressions allow us to gain insights into their equations.

In this article, we will extend their work by exploring the intrinsic characteristics of their equations and applying them to two types of copolymer originally not intended to be covered by their equations: compatible multiblock copolymers and homopolymers with various tacticities treated as steric copolymers.

# New expressions and characteristics of Barton's and Johnston's equations

Since a given copolymer is composed of two monomers, the first way for expressing the comonomer sequences is to take the fractions of diads into account. There are three different diads in a copolymer chain, AA, BB and AB (=BA),

so long as the direction of the chain is not important. With these diads, Barton and Johnston derived, respectively, the equations for the glass transition temperature of a copolymer as follows

$$T_{g} = m_{A}P_{AA}T_{gAA} + m_{B}P_{BB}T_{gBB} + (m_{A}P_{AB} + m_{B}P_{BA})T_{gAB}$$
(B) (2a)  
$$1/T_{g} = W_{A}P_{AA}/T_{gAA} + W_{B}P_{BB}/T_{gBB}$$
$$+ (W_{A}P_{AB} + W_{B}P_{BA})/T_{gAB}$$
(J) (2b)

Here,  $T_g$  is the glass transition temperature of a copolymer with composition and comonomer sequences specified by the values of products *mPs* or *WPs*;  $P_{ij}$ is the probability that a given *I* monomer is followed by a *J* monomer;  $T_{gIJ}$  is the  $T_g$  value of the polymer composed of *IJ* diads only. That is, the product of  $m_A P_{AA}$  is equal to the mole fraction of the AA diad,  $m_{AA}$ ; similarly,  $m_B P_{BB} = m_{BB}$  and so on.  $T_{gAA}$  and  $T_{gBB}$  are the  $T_g$ s of the respective homopolymers and  $T_{gAB}$  is the  $T_g$  of the corresponding strictly alternating copolymer.

The parameter  $m_{IJ}$  can be defined, after Barton, as the fraction of rotatable bonds in diad IJ. For simplicity, that is taken to be the mole fraction of diad IJ in this article. This means that the number of possible rotatable bonds in A monomer unit is assumed to be the same as that in B monomer unit and the polymerization process produces only rotatable bonds.

Since a given A monomer must be followed by either A or B, the fraction of AA diads is equal to the fraction of A monomer minus the fraction of AB diads:

$$m_{\rm AA} = m_{\rm A} - m_{\rm AB} \tag{3a}$$

Similarly

$$m_{\rm BB} = m_{\rm B} - m_{\rm BA} \tag{3b}$$

We may note that every sequence of monomer A is followed by a sequence of monomer B, and there are, in a copolymer chain long enough, as many A sequences as B sequences. Those two types of sequences are linked by an AB or BA diad. So  $m_{BA}$  is equal to  $m_{AB}$ . Let us express the number of AB and BA bonds in the copolymer chains as that per 100 monomer units. This is called the run number  $\overline{R}$  [4], which is equal to the percentage of sum of AB and BA diads:

$$R = 100 (m_{\rm AB} + m_{\rm BA}) = 200 m_{\rm AB}$$
(4)

From Eqs 3a and 4, one obtains

$$m_{\rm AA} = m_{\rm A} - \overline{R}/200 \tag{5}$$

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This equation explicitly shows that a diad fraction has information on both composition and comonomer sequences. Substitution of Eq. (5) into Eq. (2a) yields [5, 6]

$$T_{\rm g} = m_{\rm A}T_{\rm gAA} + m_{\rm B}T_{\rm gBB} + (\overline{R}/100)(T_{\rm gAB} - \overline{T}_{\rm g}) \tag{6a}$$

with

 $\overline{T}_{\rm g} = (T_{\rm gAA} + T_{\rm gBB})/2$ 

Similarly, substitution of Eq. (5) into Eq. (2b) yields [6, 7]

$$1/T_{g} = W_{A}/T_{gAA} + W_{B}/T_{gBB} + (\overline{R}/200\overline{M})$$
$$\times [(M_{A} + M_{B})/T_{gAB} - M_{A}/T_{gAA} - M_{B}/T_{gBB}]$$
(6b)

with

$$\overline{M} = m_{\rm A}M_{\rm A} + m_{\rm B}M_{\rm B}$$

Here,  $M_X$  (X=A or B) stands for the formula weight of monomer X. The complicated probabilities,  $P_{IJ}s$ , no longer appear in the equations. The  $\overline{R}$  value of a given copolymer can be determined by nuclear magnetic resonance experiments [8] or calculated by

$$\overline{R} = 400 \ m_{\rm A} m_{\rm B} / \left( 1 + [1 + 4m_{\rm A} m_{\rm B} (r_{\rm A} r_{\rm B} - 1)]^{\frac{1}{2}} \right)$$
(7)

where  $r_A r_B$  is the product of the monomer reactivity ratios in copolymerization. The  $\overline{R}$  value is zero for homopolymers and 100 for alternating copolymers. Those of statistical copolymers vary with composition, having a maximum,  $\overline{R}^*$ , at  $m_A = 0.5$  for low-conversion products. Those of diblock copolymers are practically zero, for there is only one AB diad in a long chain.

Since the  $T_{gAB}$  is often unknown, another expression of Eq. (6a) should be derived. For the  $T_g$  of equimolar statistical copolymer, Eq. (6) reads

$$T_{\rm g}(m_{\rm A}=0.5) = \overline{T}_{\rm g} + (\overline{R}^{*}/100)(T_{\rm gAB} - \overline{T}_{g})$$
 (8)

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Elimination of  $T_{gAB}$  from Eqs (6a and 8) yields

$$T_{g} = m_{A}T_{gAA} + m_{B}T_{gBB} + (\overline{R} / \overline{R}^{*})[T_{g}(\overline{R}^{*}) - \overline{T}_{g}]$$
<sup>(9)</sup>

An expression of Johnston's equation, similar to Eq. (9) can be seen [6, 7]. These expressions show two facts: (i) the  $T_g$  of a copolymer is a function of two independent variables, composition ( $m_B = 1 - m_A$ ) and the number of comonomer

reversals ( $\overline{R}$ ) and (ii) the  $\overline{R}$  -dependent term is separated from the *m*-dependent ones.

The usual plot of  $T_g$  against  $m_B$  as in Fig. 1 may be considered to be a projection of the  $T_g$  curve in three dimensions onto the  $T_g-m_B$  plane. Its projection onto the  $T_g-\overline{R}$  plane, yielding a plot like Fig. 2a, is suitable to represent the dependence of  $T_g$  on  $\overline{R}$ . Moreover, certain mathematical characteristics useful for data analysis may be derived from this type of plot [5]. For Johnston's equation, similar characteristics are found when  $1/T_g$  is plotted vs.  $\overline{R}$  [6, 7].



Fig. 2 a) Schematic representation of plot of T<sub>g</sub> vs. R̄, showing two characteristics of Eq. (6a); b) Plots of T<sub>g</sub> vs. R̄ for styrene-α-methylstyrene (S/αMS) multiblock (Δ) copolymers together with predictions (--) of Eq. (6a) for statistical (0) copolymers

Those of Barton's equation are as follows. (i) The three points,  $\overline{T}_g$ ,  $T_g(R^*)$  and  $T_{gAB}$  are on the same straight line and (ii)  $T_g$  values for two statistical copolymers having a given value of  $\overline{R}$  but different compositions deviate by an equal distance from this line. It is clear that the straight line connecting  $\overline{T}_g$  and  $T_{gAB}$  represents the  $T_g$  locus of hypothetical equimolar copolymers with varying degrees of comonomer alternation. The inside of the curve in Fig. 2a is the region for blocklike copolymers: the portion of the ordinate,  $T_{gAA}$  to  $T_{gBB}$ , may be considered to represent the  $T_g$  locus of compatible diblock copolymers with varying degrees of composition. Barton's and Johnston's equations were originally derived for statistical copolymers but actually cover  $T_g$  values of (compatible) multiblock copolymers as well.

### Data analysis and discussion

#### Compatible block copolymers

In general, blocks of a given multiblock copolymer are, like two different homopolymer chains, incompatible, forming the two-phase structure and showing two distinct glass transition temperatures. In the literature, however, there are the data for styrene- $\alpha$ -methylstyrene (S/ $\alpha$ MS) block copolymers having single glass transition temperatures [9, 10]. Those  $T_g$  data appear to be suitable for analysis. The copolymers are of the multiblock type: the total number of blocks in a chain is from 2 to 17 and the  $\overline{R}$  value calculated varies from 0 to 7.2 [11]. The  $T_g$  data are plotted against  $\overline{R}$  in Fig. 2b.

In this figure, the  $T_g$  data of the corresponding statistical copolymers and homopolymers [12] are also included and used to construct the relationship according to Barton's equation. The other necessary data are the values of  $r_A r_B$ and  $T_{gAB}$ . The former is 0.705 [12] and the latter can be estimated to be 406.1 K from the slope,  $(\overline{T}_g - T_{gAB})/100$ , of a plot of  $(m_A T_{gAA} + m_B T_{gBB} - T_g)$  against  $\overline{R}$ .

As is deduced above, it can be seen from Fig. 2b that the  $T_g$  points of multiblock copolymers are located inside the theoretical  $T_g$  curve for the corresponding statistical copolymers. Actually, they are in the vicinity of the ordinate because their  $\overline{R}$  values are small and close to each other. However, their  $T_g$  values differ to a considerable extent, reflecting the differences in composition. Therefore, it is possible to calculate, by use of a lever rule, the composition of a sample from its  $T_g$  value together with two  $T_g$  values on the theoretical curve at the  $\overline{R}$  value of the sample.

For example, the  $T_g$  value of diblock copolymer with  $m_s = 0.53$  is 412.2 K. The two  $T_g$  values on the curve at  $\overline{R} = 0$  are 380 K ( $m_s = 1.0$ ) and 453 K ( $m_s = 0$ ) [12]. In order to check on consistency, the composition of this diblock copolymer can be estimated from the three  $T_g$  values to be  $m_s = 0.56$ . This is in good accord with the experimental value. Similar agreement can be seen for the data of the multiblock copolymers as well [11]. The new expressions of Johnston's equation can also be used for the sample purpose.

### Homopolymers with various tacticities

According to Bovey [13], tacticity is represented by the relative configuration of pairwise units, meso (m) and racemo (r). A part of a given structure is shown below.



Three types of sequences which can be seen above, mm, mr and rr, are called, respectively, isotactic (I), heterotactic (H) and syndiotactic (S) triads. These triads correspond to AA, AB and BB in ordinary copolymers. So, like comonomer arrangement, homopolymer tacticity can be treated by the statistics of reaction probability [14].

Homopolymers including these triads may be called steric copolymers. Equations 2a and b can be applied to such homopolymers by simply replacing AA with mm, i.e., I and so on.

$$T_{g} = IT_{gI} + ST_{gS} + HT_{gH} \tag{10a}$$

$$1/T_{\rm g} = I/T_{\rm gI} + S/T_{\rm gS} + H/T_{\rm gH}$$
 (10b)

Here,  $T_g$  is the  $T_g$  value of homopolymer having the tacticity specified by *I*, *S* and *H*.  $T_{gK}$  (*K*=*I*, *S* or *H*) is the  $T_g$  value of pure *K*-tactic form. The mole fraction of *I* triad is written as *I* itself as usual, the sum of *I*, *S* and *H* being unity. It is not necessary to distinguish between weight and mole fractions in this case. With these equations, the dependence of  $T_g$  of poly(methyl methacrylate) (PMMA) on tacticity will be considered. Before applying them to  $T_g$  data, let us derive new expressions of Eqs (10a and b). Since

$$m = I + H/2$$
 and  $r = S + H/2$ 

Equations (10a and b) can be written as

$$T_{g} = (1 - r)T_{gI} + rT_{gS} + H[T_{gH} - (T_{gI} + T_{gS})/2]$$
(11a)

$$1/T_{\rm g} = (1 - r)/T_{\rm gI} + r/T_{\rm gS} + H[1 / T_{\rm gH} - (1/T_{\rm gI} + 1/T_{\rm gS})/2]$$
(11b)

These expressions correspond, respectively, to Eqs (6a and b). In order to determine the dependence of glass transition temperature on tacticity, plots of  $T_g$  vs. r (=1-m) have repeatedly been made. Equation 11a implies, however, such a plot cannot be linear: the deviations from the linear relation are always expected due to the third term. As the three  $T_g$ s are constant, the deviation is seen to be proportional to the value of H in each sample. So it is not sensible to make an ordinary plot of  $T_g$  against r.

In Eq. (10)  $T_g$ , *I*, *S* and *H* are observables. In addition, both  $T_{gI}$  and  $T_{gS}$  can be determined, for it is possible to prepare PMMA molecules in almost pure tactic forms. The following values are used below: 315.2 K for  $T_{gI}$  and 400.2 K for  $T_{gS}$ . So the only unknown is  $T_{gH}$ , and  $T_g$  value of the heterotactic polymer specified by the sequence of strict alternation of *m* and *r*. This quantity can be estimated from the plot shown in Fig. 3a. This plot is based on Eq. (10b): the slope of the linear line passing through the origin is the reciprocal of  $T_{gH}$ . The used  $T_g$  data of various types of PMMAs prepared with various catalysts are listed in Table 1 [15–18]. From this plot, the  $T_{gH}$  was estimated to be 400.1 K. Similarly, a plot based on Eq. (10a) yields a value of 390.7 K for  $T_{gH}$ .

Now the experimental  $T_g$  values can be compared with those calculated by Eq. (10). These are given in the last two columns of Table 1. Good agreement

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Sample	Triads/%		T <sub>g</sub> /K		
	I	S	obs.	Eq. (10a)	Eq. (10b)
1*	99	0	315.9	315.9	315.9
2	81	5	336.7	329.7	328.3
3	80	7	336.2	330.7	329.0
4	67	15	344.5	341.2	338.7
5	62	17	340.6	345.1	342.6
6	57	24	350.3	349.6	346.6
7	38	30	364.2	364.2	362.5
8	28	36	372.3	372.2	371.5
9*	7	62	389.9	390.7	392.2
10*	7	62	391.7	390.7	392.2
11*	4	65	397.5	393.2	395.4
$12^{*}$	0	94	398.8	399.5	400.1
1 <b>3</b> *	0	94	396.6	399.5	400.1
14*	6	43	388	390.8	395.0
15*	- 3	65	394	393.9	396.4

 Table 1 Tacticity data and observed and predicted glass transition temperatures of various types of PMMA samples

\* Close to Bernoullian and denoted by solid circles in the figure.



Fig. 3 a) T<sub>gH</sub> determination based on Eq. (10b) and two T<sub>g</sub> values: T<sub>g1</sub>=315.2 K and T<sub>gS</sub>=400.2 K. (●) close to Bernoullian and (o) deviated from Bernoullian PMMAs;
b) Plot of T<sub>g</sub> against S for various types of PMMAs together with theoretical predictions by Eq. (10b) for Bernoullian (—) and stereoblock (-·-) PPMAs

is seen, with some exceptions, for all the samples, regardless of the differences in preparation. This means that there is a one-to-one correspondence between the  $T_g$  of PMMA and a set of its triads. The exceptions are isotactic samples 2 to 5. Since  $T_{gI}$  is markedly lower than  $T_{gS}$  and  $T_{gH}$  and the latter are more or less equal, it is clear that errors in the determination of tacticity have a bigger influence on the  $T_g$  values of isotactic samples.

It was understood that Eq. (10) can be applied to describing the dependence of  $T_g$  of homopolymers on tacticity. These  $T_g$  data are plotted against S in Fig. 3b. Before looking at the experimental points, we will consider theoretical predictions for some cases. A solid line, based on Eq. (10b), is for a simple case of polymerization kinetics, assuming that the configuration of an adding monomer unit is independent of that of the growing chain. These polymers are called Bernoullian, whose triad fractions can be expressed as

$$I = (1 - r)^2, \quad H = 2r(1 - r), \quad S = r^2$$
 (12)

Thus, the solid line in Fig. 3b is the  $T_g$  locus of Bernoullian PMMAs. Dash-dot lines and the ordinate are the boundaries: I=0, H=0 and S=0. The dash-dot lines for I=0 and H=0 represent, respectively, the  $T_g$  loci of H-S and I-S stereoblock polymers with varying degrees of S. The  $T_g$  data of all types of PMMAs must be inside the triangle defined by these three line segments. The data are located in the triangle, but they do not fit a single curve. Filled circles are close to the solid line, while open circles are not.

This is quite natural. A big difference between tactic polymers like PMMAs and ordinary statistical copolymers must be recognized. The comonomer sequences of the latter is governed, as can be seen from Eq. (7), by a single parameter  $r_A r_B$ . It is impossible, however, to prepare PMMA molecules with various tacticities under specific polymerization kinetics. We recall the finding of Bovey and Tiers that free radical PMMA molecules are Bernoullian but anionic PMMA molecules are not [19]. This figure confirms their finding only in a different way.

In this discussion, we tried to understand the relationship of glass transition temperature of a copolymer with its comonomer sequences. As a theoretical basis, Barton's and Johnston's equations were used. Although they are known to be useful for ordinary copolymers, both equations were found to be applicable to compatible multiblock copolymers and steric copolymers like PMMA, providing further understanding of the  $T_{gs}$  of these types of copolymers.

So far, the glass transition was implicitly assumed to be a property determined primarily by intramolecular interactions. And we did not ask why, for instance,  $T_{gAB}$  becomes smaller than  $T_g(\overline{R}^*)$  in the case shown in Fig. 1. An answer could be found from a consideration based on the actual molecular structure of comonomer sequences. Such conclusions have already been published, for several copolymer systems including tactic PMMAs, by Tonelli [20]. The question of which theory is more valid, Barton's (configurational entropy theory) or Johnston's (free-volume theory), still remains unsettled, however. This problem is now under intensive investigation by use of the copolymer systems whose  $T_{gAB}$  values predicted are markedly different from each other [6].

\* \* \*

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